Performance Comparison between Dollar Cost Averaging and Value Averaging Investment Strategies and the Impacts of Investment Horizon and Target Terminal Wealth

Kamphol Panyagometh[1]

Abstract

This study employs the techniques of Monte Carlo Simulation and Genetic Algorithms Based Optimization aimed at analyzing the impacts of investment horizon and target terminal wealth on the performance of the Dollar Cost Averaging (DCA) investment strategy in comparison with the Value Averaging (VA) investment strategy. According to the findings, with increased length of investment horizon and/or lowered target terminal wealth, the Value Averaging (VA) investment strategy will have better performance than the Dollar Cost Averaging investment strategy. The investment performance is evaluated with a variety of measures including Modified Sharpe Ratio, Modified Sortino Ratio, Shortfall Probability and Dominance Probability.

1   Introduction

Dollar Cost Averaging (DCA) is one of the most popular investment strategies recommended by financial planners and investment advisors, even though there are numerous academic articles indicating that the DCA investment strategy is less efficient than other investment strategies such as the Lump Sum (LS) or Value Averaging (VA) investment strategies. Constantinides [1] demonstrated that, in theory, the DCA is a

[1]PhD, CFA, FRM, CFP, NIDA Business School, Bangkok, Thailand
e-mail: kpanyagometh@gmail.com

suboptimal investment strategy. Brennan and Solanki [2] explained that the DCA is suboptimal when the returns from securities were independently and identically distributed and the investor's objective is to maximize expected utility of terminal wealth. Williams and Bacon [3] and Rozeff [4] studied the annual returns from the DCA strategy in comparison with the LS strategy and concluded the LS strategy to outperform the DCA strategy. Brennan, Li, ad Torous [5] asserted that the DCA would outperform the LS strategy when the securities had a pricing pattern in the form of mean reversion, while Greenhut [6] concluded that the DCA would outperform the LS when the stock exchange is declining. Regarding the comparison between the DCA and VA investment strategies, Marshall [7] and Leggio and Lien [8] found the DCA to underperform the VA.   Chen and Estes [9] and Chen and Estes [10] employing Monte Carlo Simulation in order to study investment strategies for investors in the 401(k) plan in the United States of America by comparing the DCA and the VA investment strategies, found the VA strategy to outperform the DCA strategy when the target annual growth rate of the VA was from 8% to 12%.

Although previous research shows the VA to outperform the DCA, previous studies have not clearly demonstrated whether the performance of the VA strategy is better or worse than the DCA, which depends upon the investment horizon and the target terminal wealth. The objective of this study is to emphasize that both of these factors are significant to the performance of the DCA strategy comparing with the VA strategy. Section 2 will address the data used in the research and Section 3 will discuss the DCA and VA investment strategies to be compared in this study. Section 4 states the research findings and Section 5, Conclusion, summarizes the research findings.

2   Research Data

This study used monthly data between March 2000 to November 2010 from the SET Total Return Index (SET TRI) calculated by the Stock Exchange of Thailand (SET), which is representative of investments in common stock assets and the TBMA Government Bond Total Return Index, BOND TRI calculated by the Thai Bond Market Association, which is representative of investments in government bond assets.

According to data from the SET TRI and the BOND TRI during the aforementioned period, the total average annual returns from investments in the SET TRI and the BOND TRI were equal to 12.59% and 5.73%, respectively. The standard deviation for the annual returns from investments in the SET TRI and the BOND TRI equaled 25.59% and 6.26% respectively. And the correlation coefficient between common stock and debt securities was equal to -0.086. The average annual return and standard deviation of the annual rate of return and the correlation coefficient from investments in the aforementioned SET TRI and BOND TRI will be used as inputs in performing the Monte Carlo Simulation according to the studies of Chen and Estes [9], Chen and Estes [10], Abeysekera and Rosenbloom [11] and Marshall [7].

Table 1: Terminal Wealth from Monte Carlo Simulation

Lenghts of pre-retirement investment period	Mean Terminal Wealth		SD Terminal Wealth	
	Bonds	Stocks	Bonds	Stocks
35	11,550,242	67,741,503	3,147,853	130,540,927
30	8,229,695	35,682,721	2,044,952	57,782,355
25	5,727,790	19,027,149	1,254,173	27,863,475

Table 1 demonstrates the terminal wealth from the Monte Carlo Simulation from an investment of one baht at the beginning of every year in cases where 100% was invested in the SET TRI and in cases where 100% was invested in the BOND TRI. According to the lengths of pre-retirement investment horizon at twenty-five, thirty and thirty-five years for investments in the SET TRI, if we invest one baht in the SET TRI at the beginning of every year with an investment horizon at twenty-five years before retirement, Table 1 shows that the investment portfolio will have a mean terminal wealth equal to 19,027,149 baht and a standard deviation of the terminal wealth equal to 27,863,475 baht, while cases of investment in the BOND TRI will yield a mean terminal wealth equal to 5,727,790 baht and a standard deviation of the terminal wealth equal to 1,254,173 baht.

Table 1 shows the mean terminal wealth to increase when the length of pre-retirement investment horizon increases. In terms of risk, however, the standard deviation for the rate of return from the portfolio decreases while the investment horizon increases (Panyagometh [12]; Strong and Taylor [13]; Hickman, Hunter, Byrd, Beck, and Terpening [14]). Table 1 shows the standard deviation of the terminal wealth to increase when the investment horizon during the period before retirement increases. Therefore, as Chen and Estes [10] stated, when the investment horizon during the period before retirement increases, the risk from the investment portfolio will decrease if and only if we view it in terms of the return. However, when we view in terms of the target terminal wealth, the risk of having less money than anticipated at retirement is greater. In this study, therefore, we will assess risks by looking at the terminal wealth as in the research of Chen and Estes [10] and Chen and Estes [9].

3   Dollar Cost Averaging and Value Averaging Strategies

3.1 Dollar Cost Averaging

Dollar Cost Averaging (DCA) is one of the investment strategies that financial planners and investment advisors most recommend to clients because it is an easy strategy to follow and promotes investment discipline by emphasizing regular investment regardless of the stock exchange market's direction, which enables professional investors to avoid the impacts of emotional sensitivity toward investment. When the DCA investment strategy is used, investors use the same amount every time, e.g. they invest 10,000 baht at the beginning of every month or 120,000 baht at the beginning of every year, and they do so until retirement. The money invested during each period will be divided for investment in various assets in optimal ratios according to each investor's risk tolerance and the target terminal wealth. Chen and Estes [9] studied DCA in cases where investors invested $1,000 at the beginning of every month for a period of three hundred and sixty months, or

thirty years in which 70% of the $1,000 is invested in common stocks and the remaining 30% is invested in debt securities. Chen and Estes [9] stated that a ratio of 70% in common stocks and 30% in debt securities is an optimal portfolio for investors who are able to accept risks to a moderate degree and with an investment horizon of thirty years as the period of time generally used in other previous studies (Cooley, Hubbard and Walz [15]; Stout and Mitchell [16]; Ervin, Faulk and Smolira [17]). Apart from this common stock-debt securities ratio at 70% and 30%, Chen and Estes [9] also studied DCA and VA in cases where the common stock-debt securities ratio was 60% and 40% with the investment horizon remaining at thirty years.

In this study, rather than using a common stock-debt securities ratio of 70%-30% as the only ratio and an investment horizon of thirty years, we have determined the optimal common stock-debt securities ratio for the DCA strategy by considering investment horizon, the amount of money invested before retirement, remaining life time and the amount of money required after retirement. Consider the case of Mrs. Somsri who is currently thirty-five years of age and plans to retire at the age of sixty years. Therefore, Mrs. Somsri will have a period of twenty-five years before retirement. And according to the data of the World Health Organization in 2010, the average lifespan of Thai men and women is sixty-six and seventy-four years, respectively (http://www.who.int/gho/countries/tha.pdf). Therefore, Mrs. Somsri can expect to live for approximately another fifteen years following retirement. Mrs. Somsri expects to invest 100,000 baht in her retirement at the beginning of every year over a period of twenty-five years until she retires. Mrs. Somsri would like to have money to spend every year after retirement at ten times the amount she invested before her retirement. In the case of Mrs. Somsri, that amount is one million baht in order to maintain her lifestyle. To be conservative, this study assumed the rate of return during the period following retirement to be equal to zero. Hence, Mrs. Somsri must have a total of 15,000,000 baht (one million baht per year for fifteen years) in order to be able to withdraw this amount of money to use one million baht per year for fifteen years. We call this monetary amount of fifteen million baht at retirement the minimum acceptable target wealth. In this study, we also consider the case where investors would like to have money to spend every year during their retirement at fifteen and twenty times the amount of money invested during the period before retirement, or cases of minimum acceptable target wealth equaling 22,500,000 baht and 30,000,000 baht, respectively.

This study used the RISKOptimizer program[2] in order to perform Portfolio optimization in consideration of the uncertainty of the rate of return in the future which will make the outcome achieved more reasonable. The RISKOptimizer program is combined with the technique of Simulation and Optimization in order to enable us to perform Optimization for an issue with uncertain variables in the model by using Genetic Algorithms based optimization and Monte Carlo simulation. We were able to use RISKOptimization in analyzing for a suitable answer to problems which cannot be answer with ordinary linear and non-linear optimization programs, such as the "Solver" function in Excel.

The optimal investment ratio for the DCA strategy was obtained by resolving the optimization issue as follows:

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[2]RISKOptimizer developed by Palisade Corporation; see details at http://www.palisade.com

$$\underset{w_{BONDTRI}, w_{SETTRI}}{\text{Min}} \quad \text{Downside Risk} \tag{1}$$

Subject to:

$$T\overline{W} \geq \text{Minimun Acceptable TW}$$

$$w_{BONDTRI} + w_{SETTRI} = 1$$

$$w_{BONDTRI} \geq 0$$

$$w_{SETTRI} \geq 0$$

Where:

$T\overline{W}$ = Mean Terminal Wealth

Minimum Acceptable TW = Minimum acceptable target wealth

$$\text{Downside Risk} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(\text{Minimum Acceptable TW} - TW_i \right)^2}$$

$w_{BONDTRI}$ = Optimal Ratio in BOND TRI

w_{SETTRI} = Optimal Ratio in SET TRI

Table 2 shows the SET TRI and BOND TRI ratios in optimal investment portfolios under various the investment horizons and minimum acceptable target wealths. Consider the case of Mrs. Somsri who has a twenty-five year period of investment before retirement and needs to spend ten times of her annual investment before retirement. Table 2 demonstrates that the optimal investment portfolio for Mrs. Somsri's retirement is a 73.1% investment in the SET TRI and a 26.9% investment in the BOND TRI. According to Table 1, we can see that when the investment horizon equals twenty-five years, the mean terminal wealth for investments of 100% in common stocks equals 19,027,149. Hence, Table 2 shows that when the investment horizon equals twenty-five years and the amount of money required for use per year after retirement in an amount fifteen and twenty times the amount of money invested per year during the period before retirement, or equal to the minimum acceptable target wealth of 22.5 million baht and 30.0 million baht respectively, which cannot be achieved because, even though 100% was invested in common stock, the mean terminal wealth remains lower than the minimum acceptable target wealth required. Table 2 shows the optimal ratio in common stocks to increase when the investment horizon is reduced and/or the minimum acceptable target wealth increases.

Table2: Optimum Weights for DCA and Target Return for VA under Various the
Investment Horizons and Minimum Acceptable Target Wealth

		Lenghts of pre-retirement investment horizon: 35 years		
	Minimum Acceptable	Optimal Weight		Target Return
Times	Target Wealth	Bonds	Stocks	for VA
10	15,000,000	94.0%	6.0%	18.61%
15	22,500,000	81.0%	19.0%	15.63%
20	30,000,000	67.6%	32.4%	14.63%
		Lenghts of pre-retirement investment horizon: 30 years		
	Minimum Acceptable	Optimal Weight		Target Return
Times	Target Wealth	Bonds	Stocks	for VA
10	15,000,000	75.5%	24.5%	15.98%
15	22,500,000	48.4%	51.6%	14.34%
20	30,000,000	20.7%	79.3%	13.65%
		Lenghts of pre-retirement investment horizon: 25 years		
	Minimum Acceptable	Optimal Weight		Target Return
Times	Target Wealth	Bonds	Stocks	for VA
10	15,000,000	26.9%	73.1%	13.94%
15	22,500,000	N/A		
20	30,000,000			

3.2 Value Averaging

For the Value Averaging investment strategy, investors need to set a target growth rate which will enable investors to achieve the desired minimum acceptable target wealth based upon an optimal common stock-debt securities ratio. For example, Mrs. Somsri invested 100,000 baht at the beginning of every year with the optimal investment ratio of 73.1% in common stock and 26.9% in debt securities. Therefore, in the case of the DCA strategy, Mrs. Somsri will invest 73,100 baht in common stock and 26,900 baht in debt securities at the beginning of every year until she retires. In the case of VA, common stock is used as the main driving force to achieve the minimum acceptable target wealth set earlier while debt securities are used as a reserve fund. The amount of money invested between each stock-debt securities investment will be adjusted to achieve the set target growth rate. For example, in the case of Mrs. Somsri who invested 73,100 baht in common stock at the beginning of the year until retirement over the next twenty-five years with a minimum acceptable target wealth of fifteen million baht, Mrs. Somsri's common stock portfolio will have a target growth rate of 13.94%.[3]

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[3]Use Excel where FV = 15,000,000; PMT = 73,100; PV = 0; NPER = 25 and TYPE = 1. When RATE is calculated, it will equal 13.94%.

Table 3: Difference between Investments Using the DCA and VA Investment Strategies

Year			VA		DCA	
			Bonds	Stocks	Bonds	Stocks
1	Beginning	Balance	26,900.0	73,100.0	26,900.0	73,100.0
1	Ending	Return	-0.68%	14.53%	-0.68%	14.53%
1	Ending	Balance	26,717.1	83,721.4	26,717.1	83,721.4
2	Beginning	Adjustment	27,331.3	72,668.7	26,900.0	73,100.0
2	Beginning	Balance	54,048.4	156,390.1	53,617.1	156,821.4
2	Ending	Return	1.24%	8.50%	1.24%	8.50%
2	Ending	Balance	54,718.6	169,683.3	54,281.9	170,151.3
3	Beginning	Adjustment	18,392.4	81,607.6	26,900.0	73,100.0
3	Beginning	Balance	73,110.9	251,290.9	81,181.9	243,251.3
3	Ending	Return	2.30%	-30.50%	2.30%	-30.50%
3	Ending	Balance	74,792.5	174,647.2	83,049.1	169,059.6
4	Beginning	Adjustment	-74,792.5	174,792.5	26,900.0	73,100.0
4	Beginning	Balance	0.0	349,439.7	109,949.1	242,159.6

Table 3 shows the differences between investments using the DCA and VA investment strategies. At the beginning of Year 1, both strategies invested in common stock and debt securities at equal amounts of 73,100 baht in common stock and 26,900 baht in debt securities. In Year 1, common stock yielded returns equaling 14.53% while debt securities yielded returns equaling -0.68%. Hence, the value of the common stock portfolios equaled 83,721.4 baht, while the value of the debt securities portfolios equaled 26,717.1 baht at the end of Year 1. In the case of the VA strategy, which had target growth rate for common stock portfolios of 13.94%, the value of common stock portfolio according to the target was 83,290.1 baht (= 73,100*(1+0.1394)) at the end of Year 1. Therefore, at the end of Year 1, the value of the common stock portfolio was higher than the target by 431.3 baht, so the amount invested by Mrs. Somsri in common stock was reduced by 431.3 baht at the beginning of Year 2, which was an investment of 72,668.7 baht in the common stock portfolio, which will cause common stock portfolio at the beginning of Year 2 to equal 156,390.1 baht, while the investment in debt securities increased by 431.3 baht, which was an investment of 27,331.3 baht in the debt securities portfolio. Hence, the debt securities portfolio at the beginning of Year 2 equaled 54,048.4 baht whereas the amount of money invested in the common stock and debt securities portfolios at the beginning of Year 2 remained at 73,100 baht and 26,900 baht, respectively, in the case of the DCA strategy.

In Year 2, the common stock yielded a return of 8.50%, while debt securities yielded a return of 1.24%. Hence, the value of the common stock portfolio at the end of Year 2 equaled 169,683.3 baht, while the value of the debt securities portfolio equaled 54,718.6 baht at the end of Year 2. The target value of the common stock portfolio was 178,190.9 baht (=156,390.1*(1+0.1394)). Therefore, at the end of Year 2, the value of the common stock portfolio was lower than the target value by 8,507.6 baht. Thus, at the beginning of Year 2, Mrs. Somsri will invest another 8,507.6 baht in common stock, thereby bringing the amount invested in common stock to 81,607.6 baht, which will put the value of the common stock portfolio at 251,290.9 baht at the beginning of Year 3, while the amount of money invested in debt securities will be reduced by 8,507.6 baht, causing the amount of

money invested in debt securities portfolios to be at 18,392.4 baht, thereby causing the value of the debt securities portfolio at the beginning of Year 3 to equal 73,110.9 baht. In the case of the DCA strategy, the amount of money invested in the common stock portfolio and debt securities portfolio at the beginning of Year 3 held steady at 73,100 baht and 26,900 baht, respectively.

4   Research Findings

The performance comparison between the DCA investment strategy and the VA investment strategy was conducted through various measurements, such as the mean terminal wealth, the modified Sharpe ratio, the modified Sortino ratio, shortfall probability and dominance probability. The impacts of investment horizon during the period before retirement and minimum acceptable target wealth on investment strategy efficiency were also analyzed.

4.1 Mean Terminal Wealth

Table 4 shows the results of the mean terminal wealth of the DCA investment strategy compared to the VA investment strategy. If investors have the length of pre-retirement investment horizon of thirty-five years and investors would like to have post-retirement spending at ten times the amount of money invested at 100,000 baht per year during the period before retirement, the minimum acceptable target wealth at retirement comes to fifteen million baht. Table 4 demonstrates that the VA investment strategy yield a mean terminal wealth according to the simulation of 19,834,234 baht, which is higher than the DCA investment strategy yielding a mean terminal wealth of 15,042,306. The mean terminal wealth of the VA is higher than the DCA with statistical significance as indicated according to the t-statistic values.

According to Table 4, if the investment horizon before retirement is thirty-five years, the mean terminal wealth of the VA will be higher than the DCA for every studied minimum acceptable target wealth. However, when the investment horizon before retirement is reduced to thirty years, the mean terminal wealth of the DCA will be higher than the VA with statistical significance with the minimum acceptable target wealth of more than or equal to 22.5 million baht Moreover, when the investment horizon during the period before retirement is reduced to twenty-five years, the mean terminal wealth of the DCA will remain higher than the VA with statistical significance, even though the minimum acceptable target wealth is just fifteen million baht.

Table 4: Performance Comparison between DCA and VA Based on Mean Terminal Wealth

			Optimal Weight		Mean Terminal Wealth		
	Minimum Acceptable	Target Return					
Times	Target Wealth	for VA	Bonds	Stocks	DCA	VA	t-Statistic
10	15,000,000	18.609%	93.99%	6.01%	15,042,306	19,834,234	63.42
15	22,500,000	15.629%	81.01%	18.99%	22,609,530	26,169,436	20.16
20	30,000,000	14.633%	67.57%	32.43%	30,025,471	31,480,429	4.48

Lenghts of pre-retirement investment horizon: 35 years

			Optimal Weight		Mean Terminal Wealth		
	Minimum Acceptable	Target Return					
Times	Target Wealth	for VA	Bonds	Stocks	DCA	VA	t-Statistic
10	15,000,000	15.981%	75.52%	24.48%	15,134,669	17,266,366	22.58
15	22,500,000	14.337%	48.39%	51.61%	22,523,032	21,975,943	-2.89
20	30,000,000	13.650%	20.75%	79.25%	30,235,196	25,245,974	-13.37

Lenghts of pre-retirement investment horizon: 30 years

			Optimal Weight		Mean Terminal Wealth		
	Minimum Acceptable	Target Return					
Times	Target Wealth	for VA	Bonds	Stocks	DCA	VA	t-Statistic
10	15,000,000	13.936%	26.88%	73.12%	15,326,252	13,892,550	-9.80
15	22,500,000				N/A		
20	30,000,000						

Lenghts of pre-retirement investment horizon: 25 years

Therefore, Table 4 shows that the VA will tend to have higher performance than the DCA when evaluated in terms of mean terminal wealth when the period before retirement is longer and/or when the minimum acceptable target wealth is reduced.

4.2   Modified Sharpe Ratio and Modified Sortino Ratio

In the previous section, we evaluated investment efficiency considering only the mean terminal wealth without taking risk into consideration.   In this section, we will evaluate investment efficiency by taking risk into consideration using indicators called the modified Sharpe ratio and the modified Sortino ratio.

As shown in Equation 2, the modified Sharpe ratio is the ratio of the difference between the mean terminal wealth and the minimum acceptable target wealth to the risk evaluated from the standard deviation of terminal wealth.

$$\text{Modified Sharpe Ratio} = \frac{\overline{TW} - \text{Minimun Acceptable TW}}{\sigma(TW_t)} \qquad (2)$$

While the modified Sharpe ratio uses the standard deviation of terminal wealth to measure risk, the modified Sortino ratio assesses risk in terms of downside risk as shown in Equation 3:

$$\text{Modified Sortino Ratio} = \frac{\overline{TW} - \text{Minimun Acceptable TW}}{\text{Downside Risk}} \qquad (3)$$

The higher modified Sharpe ratio and modified Sortino ratio show better reward-to-risk trade-off. The table 5 demonstrates performance measurement of VA investment strategy compared to DCA, evaluating from modified Sharpe ratio and modified Sortino ratio. The

result obtained is the same as in the case of the mean terminal wealth, i.e., when the investment horizon is thirty-five years, the modified Sharpe ratio and the modified Sortino ratio of VA are higher than those of the DCA strategy at every minimum acceptable target wealth studied. However, the DCA strategy will outperform the VA strategy in two cases: 1. When the investment horizon decreases to thirty years and the minimum acceptable target wealth is greater than or equal to 22.5 million baht; or 2. When the investment horizon decreases to twenty-five years and the minimum acceptable target wealth is fifteen million baht or more. In both cases, the modified Sharpe ratio and the modified Sortino ratio of the VA strategy are negative, thereby indicating that the VA strategy is incapable of yielding a mean terminal wealth higher than the minimum acceptable target wealth required by investors while the modified Sharpe ratio and the modified Sortino ratio of the DCA strategy remain positive.

Table 5: Performance Comparison between DCA and VA Based on Modified Sharpe Ratio and Modified Sortino Ratio

Lenghts of pre-retirement investment horizon: 35 years								
	Minimum Acceptable	Target Return	Optimal Weight		Modified Sharpe Ratio		Modified Sortino Ratio	
Times	Target Wealth	for VA	Bonds	Stocks	DCA	VA	DCA	VA
10	15,000,000	18.609%	93.99%	6.01%	0.0048	0.5681	0.0180	1.7225
15	22,500,000	15.629%	81.01%	18.99%	0.0035	0.2575	0.0222	0.6313
20	30,000,000	14.633%	67.57%	32.43%	0.0006	0.0747	0.0032	0.1662
Lenghts of pre-retirement investment horizon: 30 years								
	Minimum Acceptable	Target Return	Optimal Weight		Modified Sharpe Ratio		Modified Sortino Ratio	
Times	Target Wealth	for VA	Bonds	Stocks	DCA	VA	DCA	VA
10	15,000,000	15.981%	75.52%	24.48%	0.0081	0.2478	0.0431	0.5936
15	22,500,000	14.337%	48.39%	51.61%	0.0008	-0.0360	0.0037	-0.0755
20	30,000,000	13.650%	20.75%	79.25%	0.0051	-0.2502	0.0246	-0.4779
Lenghts of pre-retirement investment horizon: 25 years								
	Minimum Acceptable	Target Return	Optimal Weight		Modified Sharpe Ratio		Modified Sortino Ratio	
Times	Target Wealth	for VA	Bonds	Stocks	DCA	VA	DCA	VA
10	15,000,000	13.936%	26.88%	73.12%	0.0165	-0.1207	0.0759	-0.2426
15	22,500,000				N/A			
20	30,000,000							

4.3 Shortfall Probability and Dominance Probability

In this section, we evaluate investment strategy performance by considering shortfall probability and dominance probability. Shortfall probability is the chance that the investment will yield less terminal wealth than the desired minimum acceptable target wealth, as shown in Equation 4. Thus, investment strategies with lower shortfall probability will indicate better performance. Equation 5 shows the calculation of dominance probability, which is the calculation for the chance that the VA strategy will yield higher terminal wealth than the DCA. The higher the dominance probability, the more the VA strategy will outperform the DCA strategy.

$$\text{Shortfall Probability} = \sum_{i=1}^{N} \frac{f\,(TW_i - \text{Minimun Acceptable TW})}{N} \tag{4}$$

f(TW$_i$ – Minimum Acceptable TW) is equal to 1 when the TW is less than the minimum acceptable TW and equals to 0 when the TW is greater than or equal to the Minimum Acceptable TW and N is the number of times the simulation is performed.

$$\text{Dominance Probability} = \sum_{i=1}^{N} \frac{f\ (TW_i^{VA} - TW_i^{DCA})}{N} \tag{5}$$

f (TW$_i^{VA}$ – TW$_i^{DCA}$) equals to 1 when the terminal wealth from the VA strategy is higher than the terminal wealth from the DCA strategy, and equals to 0 when the terminal wealth from the VA strategy is less than or equal to the terminal wealth from the DCA strategy and N is the number of times the simulation is performed.

When shortfall probability is considered, Table 6 indicates that the VA strategy has a lower shortfall probability than the DCA strategy in every case studied, and the shortfall probability is found to be higher when the investment horizon before retirement declines and/or target terminal wealth increases. When investment performance is evaluated in terms of dominance probability, Table 6 indicates more than 60% chance that VA strategy will yield higher terminal wealth than the DCA strategy in every case studied. Furthermore the chance that the VA strategy will outperform the DCA strategy is higher when the investment horizon before retirement increases and/or target terminal wealth decreases.

Table 6: Performance Comparison between DCA and VA Based on Shortfall Probability and Dominance Probability

			Optimal Weight		Shortfall Probability		
Times	Minimum Acceptable Target Wealth	Target Return for VA	Bonds	Stocks	DCA	VA	Dominance Probability
Lenghts of pre-retirement investment horizon: 35 years							
10	15,000,000	18.609%	93.99%	6.01%	65.84%	30.54%	78.07%
15	22,500,000	15.629%	81.01%	18.99%	74.27%	43.41%	72.24%
20	30,000,000	14.633%	67.57%	32.43%	74.35%	51.86%	68.93%
Lenghts of pre-retirement investment horizon: 30 years							
10	15,000,000	15.981%	75.52%	24.48%	71.88%	44.11%	71.39%
15	22,500,000	14.337%	48.39%	51.61%	72.05%	56.86%	65.04%
20	30,000,000	13.650%	20.75%	79.25%	71.97%	65.69%	61.94%
Lenghts of pre-retirement investment horizon: 25 years							
10	15,000,000	13.936%	26.88%	73.12%	69.66%	61.00%	61.72%
15	22,500,000			N/A			
20	30,000,000						

5   Conclusion

This study examined investment strategies for retirement which financial planners and investment advisors commonly recommend to their clients, namely, the DCA investment strategy compared to the VA investment strategy. While previous academic articles point

out that the VA strategy is more efficient than the DCA strategy. This research used the Monte Carlo Simulation and Genetic Algorithms Based Optimization techniques to show that the efficiency of these two strategies depends upon the investment horizon and target terminal wealth. When investment performance is measured by mean terminal wealth, modified Sharpe ratio and modified Sortino ratio, the findings of this study indicated that when the investment horizon is longer and/or target terminal wealth is lower, the VA strategy will be more efficient and outperform the DCA strategy. However, when assessing investment efficiency in terms of shortfall probability and dominance probability, the VA tended to outperform the DCA in every case.

This research indicates that financial planners and investment advisors need to consider both the investment horizon and target terminal wealth of clients in recommending retirement investment strategies so that optimal and efficient investment strategies can be recommended to clients.

References

[1]   Constantinides, G.M., A Note on the Suboptimality of Dallar-Cost Averaging as an Investment Policy, *Journal of Financial and Quantitative Analysis* 14, 1979, pp. 443-450.

[2]   Brennan, M. J. and R. Solanki, Optimal Portfolio Insurance, *Journal of Financial and Quantitative Analysis,* **3**, 1981, pp. 279–300.

[3]   Williams, R.E., and P.W, Bacon, Lump Sum Beats Dollar-Cost Averaging, *Journal of Financial Planning*, **6(2)**, 1993, pp. 64-67

[4]   Rozeff, M.S., Lump-Sum Investing versus Dollar-Averaging, *The Journal of Portfolio Management*, **4**, 1994, pp. 45-50.

[5]   Brennan, M.J., F. Li., and W.N. Torous., Dollar Cost Averaging, *Review of Finance*, **9**, 2005, pp. 509-535.

[6]   Greenhut, J.G., Mathematical Illusion: Why Dollar-Cost Averaging Does Not Work, *Journal of Financial Planning*, **Oct**, 2006, pp. 76-83.

[7]   Marshall, P., A Statistical Comparison of Value Averaging vs. Dollar Cost Averaging and Random Investment Techniques, *Journal of Financial and Strategic Decision*, **13**, 2000, pp. 87-99

[8]   Leggio, K.B., and D. Lien., An Empirical Examination of the Effectiveness of Dollar-Cost Averaging Using Downside Risk Performance Measures, *Journal of Economics and Finance*, **27(2)**, 2003, pp.211-223.

[9]   Chen, H., and J. Estes., A Monte Carlo study of the strategies for 401(k) plans: dollar-cost-averaging, value-averaging, and proportional rebalancing, *Financial Services Review*, **19**, 2010, pp. 95-109

[10] Chen, H., and J. Estes., Value Averaging for 401(k) Plans Makes More 'Cents' than Dollar-Cost Averaging, *Journal of Financial Planning*, **20**, 2007, pp.   56-59

[11] Abeysekera, S. and E. Rosenbloom., A Simulation Model Between Lump Sum and Dollar-Cost Averaging, *Journal of Financial Planning*, **13**, 2000, pp. 86-96

[12] Panyagometh, K., Asset Allocation, Time Diversification and Portfolio Optimization for Retirement, *Technology and Investment*, **Vol. 2 No. 2**, 2011, pp. 92-104

[13] Strong, N., N. Taylor.,Time Diversification: Empirical Tests, *Journal of Business Finance & Accounting*, **28(3/4)**, 2001, pp.263-302.

[14] Hickman, K., Hunter, H., Byrd, J., Beck, J., Terpening, W., Life Cycle Investing

Holding Periods and Risk, *The Journal of Portfolio Management*, **27**, 2001, pp. 101-111.

[15] Cooley, P., C. Hubbard, and D. Walz., A Comparative Analysis of Retirement Portfolio Success Rates: Simulation versus Overlapping Periods, *Financial Services Review*, **12**, 2003, pp.115-138

[16] Stout, G., and J. Mitchell., Dynamic Retirement Withdrawal Planning, *Financial Services Review*, **15**, 2006, pp. 117-131.

[17] Ervin, D., G. Faulk, J. Smolira., The Impact of Asset Allocation, Savings, and Retirement Horizon, Saving Rates, and Social Security Income in Retirement Planning: A Monte Carlo Analysis, *Financial Services Review*, **18**, 2009, pp. 313-331.